

Emergent two-Higgs doublet models

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We investigate origin of three features that are often assumed in analysis of two-Higgs doublet models: (i) softly broken Z_2 symmetry, (ii) CP invariant Higgs potential, and (iii) degenerated mass spectra. We extend electroweak gauge symmetry, introducing extra gauge symmetry and extra scalars, and we show that our models effectively derive two-Higgs doublet models at low energy which naturally hold the three features. We also find that the models can solve the strong CP problem.

1 Introduction

The Standard Model (SM) is elaborately constructed and succeeds in reproducing almost all experimental results so far. The model is based on a $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge theory, and the unification of the weak and electromagnetic interactions is achieved by introducing a Higgs doublet and spontaneous electroweak (EW) symmetry breaking. In 2012, the signal predicted by the SM Higgs was finally found at the LHC [1, 2], so that we are sure that the Higgs particle really exists in our nature.

On the other hand, we expect that the SM is an effective model and new physics behind the SM is also discovered near future. One reason is that the SM has several non-trivial structures. For instance, the anomaly-free conditions for the gauge symmetries are achieved by the very non-trivial charge assignment of the quarks and leptons. Besides, the Higgs couplings with the fermions are unnaturally hierarchical and we do not know the origin of the Higgs potential to trigger the EW symmetry breaking. We note that some deviations from the SM predictions in some experiments have been reported, so they also motivate new physics beyond the SM.

In fact, many extensions of the SM have been proposed so far. However, it is not so easy to consider the extensions, because of the stringent experimental constraints and the non-trivial structures of the SM. As we mentioned above, the anomaly-free conditions are satisfied miraculously, so that it is not simple to introduce new fermions charged under the SM gauge groups. Furthermore, the experimental constraints are getting stronger according to the precision measurements.

Especially, the SM is very successful in flavor physics and we know that new physics should avoid tree-level Flavor Changing Neutral Currents (FCNCs).

Two-Higgs doublet model (2HDM) is one of the simple extended models without such serious problems. In this extension, one extra Higgs doublet is introduced, and the anomaly-free conditions are the same as the SM ones. In fact, 2HDMs have been widely discussed so far motivated by the experimental anomalies, such as the muon $g - 2$ [3–15]. Besides, many Beyond Standard Models (BSMs) to solve the theoretical issues in the SM predict such an extra Higgs doublet, so that 2HDMs have been well analyzed as the low-energy effective models, although it is often difficult to derive 2HDMs effectively. This is because extra other matters generally reside at the same scale as the extra Higgs doublet in many BSMs.

Even in such a simple BSM, however, some assumptions are usually supposed to avoid the conflicts with the experimental results. For instance, there are generally tree-level FCNCs in 2HDMs, if the two Higgs doublets are not distinguishable. We usually assign a softly broken Z_2 symmetry to avoid the tree-level FCNCs, and then we only allow the minimal flavor violation (MFV) [16]. Moreover, the Higgs potential contains CP violating terms in general. This is an attractive point of 2HDM, for example, for electroweak baryogenesis scenario. Nevertheless, CP invariance is often assumed in 2HDM. In addition, the results in the precision measurements of the EW interaction should be consistent with the predictions of 2HDMs. For example, the EW precision tests suggest that ρ parameter is close to one. This requires unnaturally degenerated dimensionless couplings, namely $\lambda_4 = \lambda_5$, in 2HDMs.

These three assumptions are required by the experimental results. If there is an extra Higgs doublet in our nature, these three features would offer an important clue as to the unknown physics behind the SM. In this paper, we investigate a possible underlying theory which effectively derives the 2HDM respecting the three features at low energy. Especially, we study some models where the electroweak gauge symmetry is extended¹, and show that the models naturally have the three features often assumed. We note that 2HDMs with gauged U(1) symmetry have been proposed as the origin of the softly broken Z_2 [18–20]. This U(1) symmetry may be originated from the grand unified theory [20], but the setup is complicated and the ρ parameter is deviated from one at the tree level [18–20]. Compared to this proposal, our models in this paper are much simple and can avoid too large deviation of the ρ parameter.

The rest of this paper is organized as follows. In section 2, we briefly review the 2HDM. In

¹ Another possibility is to impose some symmetries on the Higgs potential [17].

section 3.1, we propose a model with extended electroweak gauge symmetry. In section 3.2, we derive the low energy effective theory of the model discussed in Sec. 3.1, and show that it behaves as the type-I 2HDM with softly broken Z_2 symmetry without any CP phases in the Higgs potential. The model also predicts degenerated mass spectra for the CP-odd and the charged Higgs bosons. Thus all the three conditions are automatically satisfied. In section 4, we extend the model to lead 2HDM other than the type-I 2HDM. Section 5 is devoted to our conclusion.

2 Review of two-Higgs doublet models

In this section, we review the two-Higgs doublet model with the softly broken Z_2 symmetry widely discussed. We have two Higgs fields, Φ_1 and Φ_2 , charged under $SU(2)_L \times U(1)_Y$. In general the Higgs potential at the renormalizable level is given as follows:

$$\begin{aligned} V = & m_1^2 \Phi_1^\dagger \Phi_1 + m_2^2 \Phi_2^\dagger \Phi_2 - \left(m_3^2 \Phi_1^\dagger \Phi_2 + (h.c.) \right) \\ & + \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) \\ & + \left(\frac{1}{2} \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \lambda_6 (\Phi_1^\dagger \Phi_1) (\Phi_1^\dagger \Phi_2) + \lambda_7 (\Phi_2^\dagger \Phi_2) (\Phi_1^\dagger \Phi_2) + (h.c.) \right). \end{aligned} \quad (2.1)$$

Four parameters, m_3^2 , λ_5 , λ_6 , and λ_7 , can be complex, and they are CP violating. Now, we impose a softly broken Z_2 symmetry to the Higgs fields: $(\Phi_1, \Phi_2) \rightarrow (\Phi_1, -\Phi_2)$. The Z_2 symmetry forbids λ_6 and λ_7 terms. The m_3^2 term breaks the Z_2 symmetry softly, but can shift the scalar masses. Let us define the vacuum expectation values (VEVs) of the Higgs fields as

$$\langle \Phi_1 \rangle = \frac{v_d}{\sqrt{2}}, \quad \langle \Phi_2 \rangle = \frac{v_u}{\sqrt{2}}, \quad (2.2)$$

and then the relation with the Fermi constant is

$$v_u^2 + v_d^2 = \frac{1}{\sqrt{2} G_F} \equiv v^2 \simeq (246 \text{ GeV})^2. \quad (2.3)$$

We also define β as follows:

$$\cos \beta = \frac{v_d}{v}, \quad \sin \beta = \frac{v_u}{v}. \quad (2.4)$$

We have eight degrees of freedom in the scalar fields and three of them are eaten by the gauge bosons, and thus we have five physical states: two of them are CP-even states (h , H^0), one is a CP-odd state (A^0), and the others are a pair of charged scalar (H^\pm). Their masses are given by the parameters in the Higgs potential. When the Z_2 symmetry is imposed, the charged and CP-odd

Table I: Z_2 assingment of the matters in 2HDM. If the model not assign the Z_2 symmetry is called type-III.

	Φ_1	Φ_2	q_L	ℓ_L	u_R	d_R	ℓ_R
type-I	+	-	+	+	-	-	-
type-II	+	-	+	+	-	+	+
type-X	+	-	+	+	-	-	+
type-Y	+	-	+	+	-	+	-
type-III	no Z_2 assingment						

Higgs masses satisfy

$$m_A^2 = M^2 - \lambda_5 v^2, \quad (2.5)$$

$$m_{H^\pm}^2 = m_A^2 + \frac{\lambda_5 - \lambda_4}{2} v^2, \quad (2.6)$$

where

$$M^2 = \frac{m_3^2}{\sin \beta \cos \beta}. \quad (2.7)$$

Here λ_5 is assumed to be real.

There are four different assignments of the Z_2 symmetry to fermions [21–23]. The assignments and the model names are summarized in Table I. These assignments forbid FCNCs involving neutral scalars. The physics in the each 2HDM has been widely studied, although their origins of Z_2 symmetries are unclear.

In 2HDMs, the constraint of the ρ parameter, or T parameter, is important. It depends on the scalar masses and $WW h$ -coupling ($g_{WW h} = \kappa_V g_{WW h}^{SM}$). Since the LHC result implies that the coupling is almost the same as the SM prediction, we assume $\kappa_V \simeq 1$. Then the T parameter is approximately evaluated as

$$\alpha T \simeq \frac{1}{16\pi^2 v^2} \left(m_{H^0}^2 + \frac{m_A^4 (m_{H^\pm}^2 - m_{H^0}^2)}{(m_A^2 - m_{H^\pm}^2)(m_A^2 - m_{H^0}^2)} \ln \frac{m_{H^\pm}^2}{m_A^2} - \frac{m_{H^0}^4 (m_{H^\pm}^2 - m_A^2)}{(m_{H^0}^2 - m_{H^\pm}^2)(m_A^2 - m_{H^0}^2)} \ln \frac{m_{H^\pm}^2}{m_{H^0}^2} \right). \quad (2.8)$$

We easily find that the right-hand side is vanishing when either $m_A = m_{H^\pm}$ or $m_{H^0} = m_{H^\pm}$ is satisfied [24]. Since the right-hand side should be less than $\mathcal{O}(10^{-3})$ according to the electroweak precision measurements [25], two of the masses should be highly degenerated. This degeneracy requires a parameter tuning in the Higgs potential. For example $\lambda_4 \simeq \lambda_5$ is required for $m_A \simeq m_{H^\pm}$. It is unclear whether a mechanism which makes $\lambda_4 \simeq \lambda_5$ exists or not within the 2HDM.

Table II: Quantum numbers of the Higgs and matter fields ($i=1, 2, 3$).

	H_1	H_2	ϕ_3	q_L^i	ℓ_L^i	u_R^i	d_R^i	e_R^i
$SU(2)_0$	2	1	2	2	2	1	1	1
$SU(2)_1$	2	2	1	1	1	1	1	1
$U(1)_2$	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{2}$	$\frac{2}{3}$	$-\frac{1}{3}$	-1
$SU(3)_c$	1	1	1	3	1	3	3	1

3 Emergence of two-Higgs doublet models satisfying the three conditions

In this section, we consider the extension of the SM that leads 2HDMs at low energy. Our effective 2HDM will satisfy the following three conditions to evade the stringent experimental constraints:

- (i) softly broken Z_2 symmetry is remained,
- (ii) Higgs potential is CP invariant,
- (iii) Two scalars are degenerate.

At first, we consider a model which effectively realizes the type-I 2HDM. Note that only one Higgs doublet couples with all fermions and the other Higgs doublet does not in the type-I 2HDM. Depending on the vacuum alignment of the Higgs fields, the model can also leads to the Inert 2HDM [26, 27], where the lightest neutral scalar becomes stable and a dark matter candidate.

3.1 A model with extended electroweak gauge symmetry

Now, we consider a BSM with extended gauge symmetry, based on Ref. [28]. In this model, the electroweak gauge symmetry is extended to $SU(2)_0 \times SU(2)_1 \times U(1)_2$. We introduce three scalar fields, H_1 , H_2 and ϕ_3 , for the gauge symmetry breaking. Their quantum numbers are summarized in Table II. q_L^i , u_R^i and d_R^i denote left-handed and right-handed quarks respectively. ℓ_L^i and e_R^i are left-handed and right-handed leptons. The left-handed matter fields are charged under the $SU(2)_0$ gauge symmetry.

The gauge symmetric Yukawa couplings, which generate the mass matrices for the matters, are

given by

$$\mathcal{L}^{\text{Yukawa}} = - \sum_{i,j} y_u^{ij} \widetilde{q_L^i} \phi_3 u_R^j - \sum_{i,j} y_d^{ij} \widetilde{q_L^i} \phi_3 d_R^j - \sum_{i,j} y_e^{ij} \widetilde{\ell_L^i} \phi_3 e_R^j + h.c.. \quad (3.1)$$

The scalar potential is written down at the renormalizable level:

$$\begin{aligned} V(H_1, H_2, \phi_3) = & \mu_1^2 \text{tr} \left(H_1 H_1^\dagger \right) + \mu_2^2 H_2^\dagger H_2 + \mu_3^2 \phi_3^\dagger \phi_3 \\ & + \frac{1}{2} \left(\kappa \phi_3^\dagger H_1 H_2 + (h.c.) \right) \\ & + \tilde{\lambda}_1 \left(\text{tr} \left(H_1 H_1^\dagger \right) \right)^2 + \tilde{\lambda}_2 \left(H_2^\dagger H_2 \right)^2 + \tilde{\lambda}_3 \left(\phi_3^\dagger \phi_3 \right)^2 \\ & + \tilde{\lambda}_{12} \text{tr} \left(H_1 H_1^\dagger \right) \left(H_2^\dagger H_2 \right) + \tilde{\lambda}_{23} \left(H_2^\dagger H_2 \right) \left(\phi_3^\dagger \phi_3 \right) + \tilde{\lambda}_{31} \left(\phi_3^\dagger \phi_3 \right) \text{tr} \left(H_1 H_1^\dagger \right), \end{aligned} \quad (3.2)$$

Note that H_1 is a two by two matrix charged under $\text{SU}(2)_0 \times \text{SU}(2)_1$ and is defined as the field to satisfy

$$\tau^2 H_1^* \tau^2 = H_1, \quad (3.3)$$

where τ^2 is the second Pauli matrix. The Higgs potential contains one complex parameter κ , but its phase are eliminated by a field redefinition of H_2 . Therefore we can take all the parameters in the Higgs potential as real numbers. We can also take the VEVs of the Higgs fields as real thanks to the gauge symmetries. Hence we do not have any source of the CP violation in the Higgs sector.

When the heavy gauge bosons are extremely heavy, we can integrate them out and construct a low-energy effective model. The effective model is similar to the two-Higgs doublet models with softly broken Z_2 symmetry. In the followings, we discuss the effective model and see that it can be interpreted as the type-I 2HDM or the Inert 2HDM satisfying our three conditions, (i)-(iii).

3.2 Emergence of the type-I two-Higgs doublet model

Here, we discuss our effective model predicted by the BSM with the extended gauge symmetry. There are two $\text{SU}(2)$ gauge symmetries, $\text{SU}(2)_0 \times \text{SU}(2)_1$, and one scalar, H_1 , charged under the both symmetries. Then, the VEV of H_1 breaks down the symmetries to $\text{SU}(2)_L$:

$$\text{SU}(2)_0 \times \text{SU}(2)_1 \rightarrow \text{SU}(2)_L.$$

We parametrize H_1 ,

$$H_1 = \frac{1}{2}(v_1 + h_1)U_1, \text{ where } U_1 = \exp \left(i \frac{\tau^a \pi_1^a}{v_1} \right). \quad (3.4)$$

Here v_1 is the VEV of H_1 . We choose appropriate parameters in the Higgs potential to realize non-zero VEVs. According to the symmetry breaking, Nambu-Goldstone (NG) bosons appear and are eaten by the $SU(2)$ symmetry orthogonal to $SU(2)_L$.

We define the gauge fields for $SU(2)_0 \times SU(2)_1 \times U(1)_2$ as

$$\mathcal{L}^{\text{gauge}} = -\frac{1}{4} \sum_{a=1}^3 W_{0\mu\nu}^a W_0^{a\mu\nu} - \frac{1}{4} \sum_{a=1}^3 W_{1\mu\nu}^a W_1^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}, \quad (3.5)$$

where the field strengths are depicted by the gauge fields, $W_{0\mu}$, $W_{1\mu}$ and B_μ of $SU(2)_0 \times SU(2)_1 \times U(1)_2$: $W_{0\mu\nu} = \partial_\mu W_{0\nu} - \partial_\nu W_{0\mu} + ig_0[W_{0\mu}, W_{0\nu}]$, and $W_{1\mu\nu} = \partial_\mu W_{1\nu} - \partial_\nu W_{1\mu} + ig_1[W_{1\mu}, W_{1\nu}]$, and $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$, respectively.

The nonzero VEV of H_1 generates the mass term of the broken gauge symmetry, which is a linear combination of $W_{0\mu}$ and $W_{1\mu}$, and π_1^a in U_1 are eaten by the gauge field.

For convenience, let us redefine the gauge field, W_1^μ , as

$$V_1^\mu = U_1 W_1^\mu U_1^\dagger + \frac{1}{ig_1} U_1 \partial_\mu U_1^\dagger. \quad (3.6)$$

V_1^μ is transformed by the $SU(2)_0$ gauge symmetry with the gauge coupling of $SU(2)_1$, depicted by g_1 . It is useful to use V_1^μ instead of W_1^μ in this analysis. Then, we take the linear combinations of W_0^μ and V_1^μ :

$$\rho_\mu = cW_0^\mu - sV_1^\mu, \quad (3.7)$$

$$W_\mu = sW_0^\mu + cV_1^\mu, \quad (3.8)$$

where the mixing is given by

$$c = \frac{g_0}{\sqrt{g_0^2 + g_1^2}}, \quad s = \frac{g_1}{\sqrt{g_0^2 + g_1^2}}. \quad (3.9)$$

In the effective model, the gauge symmetry is $SU(2)_L \times U(1)_Y$, and W_μ is the gauge boson of $SU(2)_L$, which is given by $SU(2)_0$ with a different gauge coupling. Note that ρ^μ is a massive $SU(2)_L$ triplet. Let us define the gauge couplings in the effective model:

$$g = \frac{g_0 g_1}{\sqrt{g_0^2 + g_1^2}}, \quad g_\rho = \sqrt{g_0^2 + g_1^2}, \quad (3.10)$$

where g is the $SU(2)_L$ gauge coupling associated with W_μ . Based on the above definitions, we find

the correspondence between the fields in the original and in the effective:

$$W_1^{\mu\nu} = U_1 V_1^{\mu\nu} U_1^\dagger, \quad (3.11)$$

$$W_0^{\mu\nu} = s W^{\mu\nu} + c(D^\mu \rho^\nu - D^\nu \rho^\mu) + i g_0 c^2 [\rho^\mu, \rho^\nu], \quad (3.12)$$

$$V_1^{\mu\nu} = c W^{\mu\nu} - s(D^\mu \rho^\nu - D^\nu \rho^\mu) + i g_1 s^2 [\rho^\mu, \rho^\nu], \quad (3.13)$$

$$\hat{D}_\mu H_2 = U_1^\dagger (D_\mu \phi_2 - i g_1 s \rho_\mu \phi_2), \quad (3.14)$$

$$\hat{D}_\mu \phi_3 = D_\mu \phi_3 + i g_0 c \rho_\mu \phi_3, \quad (3.15)$$

$$\hat{D}_\mu \psi_L = D_\mu \psi_L + i g_0 c \rho_\mu \psi_L. \quad (3.16)$$

Here, ϕ_2 is defined as

$$\phi_2 = U_1 H_2, \quad (3.17)$$

\hat{D}_μ is the covariant derivative with respect to $SU(2)_0 \times SU(2)_1 \times U(1)_2$, and D_μ is the covariant derivative with respect to $SU(2)_L \times U(1)_2$. Namely, D_μ contains only W_μ and B_μ :

$$D_\mu \psi = (\partial_\mu + i g W_\mu + i g_2 Y B_\mu) \psi, \quad (3.18)$$

$$D_\mu \rho_\nu = \partial_\mu \rho_\nu + i g [W_\mu, \rho_\nu]. \quad (3.19)$$

ψ_L is the left-handed SM fermions charged under $SU(2)_0$ ($SU(2)_L$). ψ_R is the right-handed.

Finally, we find our effective Lagrangian as follows,

$$\mathcal{L} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} - V(h_1, \phi_2, \phi_3), \quad (3.20)$$

where $\mathcal{L}^{(0)}$, $\mathcal{L}^{(1)}$, and $\mathcal{L}^{(2)}$ are given by

$$\begin{aligned}\mathcal{L}^{(0)} = & + \bar{\psi}_L i \gamma^\mu D_\mu \psi_L + \bar{\psi}_R i \gamma^\mu D_\mu \psi_R - \bar{\psi}_L \tilde{\phi}_3 y_u \psi_R - \bar{\psi}_L \phi_3 y_d \psi_R + (h.c.) \\ & + \frac{1}{2} \partial_\mu h_1 \partial^\mu h_1 + D_\mu \phi_2^\dagger D_\mu \phi_2 + D_\mu \phi_3^\dagger D_\mu \phi_3 \\ & - \frac{1}{2} \text{tr}(W_{\mu\nu} W^{\mu\nu}) - \frac{1}{4} B_{\mu\nu} B^{\mu\nu},\end{aligned}\tag{3.21}$$

$$\begin{aligned}\mathcal{L}^{(1)} = & - g_0 c \bar{\psi}_L \gamma^\mu \rho_\mu \psi_L \\ & + i g_1 s \left(\phi_2^\dagger \rho_\mu D^\mu \phi_2 - (D^\mu \phi_2^\dagger) \rho_\mu \phi_2 \right) \\ & - i g_0 c \left(\phi_3^\dagger \rho_\mu D^\mu \phi_3 - (D^\mu \phi_3^\dagger) \rho_\mu \phi_3 \right),\end{aligned}\tag{3.22}$$

$$\begin{aligned}\mathcal{L}^{(2)} = & + \frac{1}{4} \rho_\mu^a \rho^{a\mu} \left(g_1^2 s^2 \phi_2^\dagger \phi_2 + g_0^2 c^2 \phi_3^\dagger \phi_3 \right) + \frac{1}{8} (v_1 + h_1)^2 g_\rho^2 \rho_\mu^a \rho^{a\mu} \\ & - i g \text{tr}([\rho_\mu, \rho_\nu] W^{\mu\nu}) \\ & - \frac{1}{2} \text{tr}((D_\mu \rho_\nu - D_\nu \rho_\mu)(D^\mu \rho^\nu - D^\nu \rho^\mu)) \\ & - i(g_0 c^3 + g_1 s^3) \text{tr}([\rho_\mu, \rho_\nu](D^\mu \rho^\nu - D^\nu \rho^\mu)) \\ & + \frac{1}{2} (g_0^2 c^4 + g_1^2 s^4) \text{tr}([\rho_\mu, \rho_\nu][\rho^\mu, \rho^\nu]).\end{aligned}\tag{3.23}$$

$V(h_1, \phi_2, \phi_3)$ is the scalar potential for the effective lagrangian, given by substituting the scalar fields to Eq. (3.2). Now, h_1 is gauge singlet, and ϕ_2 and ϕ_3 are $\text{SU}(2)_L$ -doublet. Then, we successfully derive a 2HDM with an extra singlet scalar boson and an extra $\text{SU}(2)_L$ vector boson. Note that $\text{U}(1)_2$ is identified to $\text{U}(1)_Y$ in the SM.

Now, we assume that the breaking scale of $\text{SU}(2)_1 \times \text{SU}(2)_2$ is much higher than the EW scale. We find that ρ^μ and h_1 gain the mass proportional to v_1 so that they become extremely heavy and can be integrated out much above the EW scale under this assumption. Then the effective lagrangian around the EW scale can be described as

$$\mathcal{L} = \mathcal{L}^{(0)} + V_{eff}(\phi_2, \phi_3) + (\text{higher dimensional operators}).\tag{3.24}$$

Defining the VEVs of ϕ_2 and ϕ_3 as $v_2/\sqrt{2}$ and $v_3/\sqrt{2}$, $V_{eff}(\phi_2, \phi_3)$ is expressed as below:

$$\begin{aligned}
V_{eff}(\phi_2, \phi_3) = & -\kappa \frac{v_3 v_1}{4v_2} \left(\phi_2^\dagger \phi_2 - \frac{v_2^2}{2} \right) - \kappa \frac{v_1 v_2}{4v_3} \left(\phi_3^\dagger \phi_3 - \frac{v_3^2}{2} \right) + \kappa \frac{v_1}{4} (\phi_2^\dagger \phi_3 + \phi_3^\dagger \phi_2) \\
& + \left(\tilde{\lambda}_2 + \frac{v_1^2 \tilde{\lambda}_{12}^2}{2m_{h_1}^2} \right) \left(\phi_2^\dagger \phi_2 - \frac{v_2^2}{2} \right)^2 + \left(\tilde{\lambda}_3 + \frac{v_1^2 \tilde{\lambda}_{31}^2}{2m_{h_1}^2} \right) \left(\phi_3^\dagger \phi_3 - \frac{v_3^2}{2} \right)^2 \\
& + \left(\tilde{\lambda}_{23} + \frac{v_1^2}{m_{h_1}^2} \tilde{\lambda}_{12} \tilde{\lambda}_{31} \right) \left(\phi_2^\dagger \phi_2 - \frac{v_2^2}{2} \right) \left(\phi_3^\dagger \phi_3 - \frac{v_3^2}{2} \right) \\
& + \frac{\kappa^2}{32m_{h_1}^2} \left(\phi_2^\dagger \phi_3 + \phi_3^\dagger \phi_2 - v_2 v_3 \right)^2 \\
& + \frac{\kappa v_1}{4m_{h_1}^2} \tilde{\lambda}_{12} \left(\phi_2^\dagger \phi_2 - \frac{v_2^2}{2} \right) \left(\phi_2^\dagger \phi_3 + \phi_3^\dagger \phi_2 - v_2 v_3 \right) \\
& + \frac{\kappa v_1}{4m_{h_1}^2} \tilde{\lambda}_{31} \left(\phi_3^\dagger \phi_3 - \frac{v_3^2}{2} \right) \left(\phi_2^\dagger \phi_3 + \phi_3^\dagger \phi_2 - v_2 v_3 \right). \tag{3.25}
\end{aligned}$$

We can easily find out the correspondence between the Higgs potential in Eq.(2.1) and $V_{eff}(\phi_2, \phi_3)$, according to the following identifications,

$$\phi_2 = \Phi_1, \quad v_2 = v_d, \tag{3.26}$$

$$\phi_3 = \Phi_2, \quad v_3 = v_u. \tag{3.27}$$

The each parameter in V_{eff} corresponds to the one in Eq. (2.1) as follows:

$$m_1^2 = -\frac{\kappa v_1 v_3}{4v_2} - v_2^2 \left(\tilde{\lambda}_2 + \frac{v_1^2 \tilde{\lambda}_{12}^2}{2m_{h_1}^2} \right) - \frac{v_3^2}{2} \left(\tilde{\lambda}_{23} + \frac{v_1^2 \tilde{\lambda}_{12} \tilde{\lambda}_{31}}{m_{h_1}^2} \right) - \frac{\kappa v_1 v_2 v_3}{4m_{h_1}^2} \tilde{\lambda}_{12}, \tag{3.28}$$

$$m_2^2 = -\frac{\kappa v_1 v_2}{4v_3} - v_3^2 \left(\tilde{\lambda}_3 + \frac{v_1^2 \tilde{\lambda}_{31}^2}{2m_{h_1}^2} \right) - \frac{v_2^2}{2} \left(\tilde{\lambda}_{23} + \frac{v_1^2 \tilde{\lambda}_{12} \tilde{\lambda}_{31}}{m_{h_1}^2} \right) - \frac{\kappa v_1 v_2 v_3}{4m_{h_1}^2} \tilde{\lambda}_{12}, \tag{3.29}$$

$$m_3^2 = \frac{\kappa v_1}{4} - \frac{\kappa^2 v_2 v_3}{16m_{h_1}^2} - \frac{\kappa v_1 v_2^2}{8m_{h_1}^2} \tilde{\lambda}_{12} - \frac{\kappa v_1 v_3^2}{8m_{h_1}^2} \tilde{\lambda}_{31}, \tag{3.30}$$

$$\lambda_1 = 2\tilde{\lambda}_2 + \frac{v_1^2 \tilde{\lambda}_{12}^2}{m_{h_1}^2}, \tag{3.31}$$

$$\lambda_2 = 2\tilde{\lambda}_3 + \frac{v_1^2 \tilde{\lambda}_{31}^2}{m_{h_1}^2}, \tag{3.32}$$

$$\lambda_3 = \tilde{\lambda}_{23} + \frac{v_1^2}{m_{h_1}^2} \tilde{\lambda}_{12} \tilde{\lambda}_{31}, \tag{3.33}$$

$$\lambda_4 = \lambda_5 = \frac{\kappa^2}{16m_{h_1}^2} \simeq 0, \tag{3.34}$$

$$\lambda_6 = \frac{\kappa v_1}{4m_{h_1}^2} \tilde{\lambda}_{12} \simeq 0, \tag{3.35}$$

$$\lambda_7 = \frac{\kappa v_1}{4m_{h_1}^2} \tilde{\lambda}_{31} \simeq 0. \tag{3.36}$$

The relation, $\lambda_4 = \lambda_5$, is respected, so that $m_{H^\pm}^2 = m_A^2$ is satisfied, according to Eq. (2.6). Since $m_{h_1}^2$ is $\mathcal{O}(v_1^2)$ and much heavier than the other dimensional parameters, we can conclude that λ_4 and λ_5 are very tiny. Similarly, λ_6 and λ_7 are almost vanishing, in our model. Therefore V_{eff} is the same as the Higgs potential in 2HDM with softly broken Z_2 symmetry, keeping CP symmetry. Besides, V_{eff} leads the degenerated masses for the CP-odd and the charged Higgs bosons. Then we conclude that our model naturally explains the origins of the three conditions, (i)-(iii), that are often assumed in studies of 2HDMs.²

3.3 Emergence of the Inert doublet model

In this section, we consider one specific scenario, in the framework of the Type-I 2HDM. In general, the two Higgs doublets gain non-vanishing VEVs, but, in fact, ϕ_2 need not develop a nonzero VEV because ϕ_2 does not couple with the SM fermions. We focus on the scenario with $v_2 = 0$ below.

This kind of model is called the Inert 2HDM [26, 27]. This model predicts a stable neutral particle which can be a dark matter candidate. This is an attractive feature of this model. $v_2 = 0$ is realized by $\kappa = 0$, as we see from Eq. (3.2). Then, we write down the Higgs potential in this scenario:

$$\begin{aligned}
V(h_1, \phi_2, \phi_3) = & \tilde{\lambda}_3 \left(\phi_3^\dagger \phi_3 - \frac{v_3^2}{2} \right)^2 \\
& + \left(\mu_2^2 + \frac{1}{2} \tilde{\lambda}_{12} v_1^2 \right) \phi_2^\dagger \phi_2 + \tilde{\lambda}_2 (\phi_2^\dagger \phi_2)^2 + \tilde{\lambda}_{23} (\phi_2^\dagger \phi_2) (\phi_3^\dagger \phi_3) \\
& - \frac{1}{4} \tilde{\lambda}_{31} v_3^2 h_1^2 + \frac{1}{4} \tilde{\lambda}_1 (h_1^2 + 2v_1 h_1)^2 \\
& + \frac{1}{2} (h_1^2 + 2v_1 h_1) \left(\tilde{\lambda}_{12} \phi_2^\dagger \phi_2 + \frac{1}{2} \tilde{\lambda}_{31} \phi_3^\dagger \phi_3 \right). \tag{3.37}
\end{aligned}$$

Even if h_1 is integrated out, λ_4 and λ_5 cannot be induced, as we see in Eq. (3.34). Thus $\lambda_4 = \lambda_5 = 0$ is satisfied at the tree level and this leads degenerate masses for the extra scalars of H_2 .

It is known that the degenerated CP-even and -odd neutral scalars enhance the cross section of the direct search for dark matters via the inelastic scattering of the neutral particles with nucleus through Z -boson exchanging [27]. This enhancement is enough large to exclude models. Then we have to find ways to split the masses of the two neutral scalars.

² Our model leads $m_{H^\pm} = m_A^2$. We need other models in order to lead $m_{H^\pm} = m_{H^0}^2$. Phenomenology with a light pseudoscalar in the latter condition is discussed, for example, in Ref. [29].

Naive expectation is that loop corrections split the masses. However, this does not happen due to an accidental $U(1)$ symmetry. The original Lagrangian without the κ term has an accidental symmetry under which H_2 transforms as $H_2 \rightarrow \exp(i\alpha_2)H_2$ and all the other fields do not transform. This symmetry remains even at the low energy as a global $U(1)$ symmetry only for ϕ_2 . This global symmetry allows λ_4 term, and we can expect the charged scalar mass is different from the other neutral scalar masses at loop level. On the other hand, this global symmetry forbids the λ_5 term, and thus the masses of the two neutral scalars keep degenerated even at loop level. Therefore our Inert 2HDM is excluded by the direct search for dark matters, if the relic abundance of dark matter is dominated by the neutral components of ϕ_2 .

4 Extension of the model

In the previous section, we have discussed BSMs with extended gauge symmetry which induce the type-I and the Inert 2HDMs as low-energy effective models. Based on the above discussion, we try to construct a model that leads other types of the 2HDMs at low energy.

In other types, not one but two Higgs doublet fields couple with the SM fermions, so that we cannot easily extend the gauge symmetry, under which the Higgs doublets are charged. We need some modifications of our model with $SU(2)_0 \times SU(2)_1 \times U(1)_2$. In this section, we discuss some example ways to modify and extend our model.

4.1 Emergence of the type-III two-Higgs doublet model

For the type-II, -X, -Y, and -III two-Higgs doublet models, we need more than one Yukawa interaction terms. Then, not only ϕ_3 but also H_1 and H_2 should be involved in the Yukawa interactions with the SM fermions. That is achieved if we allow the dimension-5 operators, $\bar{\psi}_L H_1 H_2 \psi_R$. This dimension-5 operators are generated if we add new vector-like fermions charged under $SU(2)_1$ gauge symmetry as shown in Table III.³ Using these new fermions and the original fermions, the

³ This setup is similar to the models discussed in Refs. [30, 31]. In those papers, the third generation in the quark sector is distinguished from the other generations.

Table III: Quantum numbers of the new fermion fields.

	Q_L	Q_R	L_L	L_R
$SU(2)_0$	1	1	1	1
$SU(2)_1$	2	2	2	2
$U(1)_2$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{2}$	$-\frac{1}{2}$
$SU(3)_c$	3	3	1	1

Yukawa interaction terms are given by

$$\begin{aligned}
\mathcal{L}^{Yukawa} = & - \sum_{i,j} \bar{q}_L^i \tilde{\phi}_3 y_u^{ij} u_R^j - \sum_{i,j} \bar{q}_L^i \phi_3 y_d^{ij} d_R^j - \sum_i \bar{\ell}_L^i \phi_3 y_e^i e_R^i \\
& - \sum_{i,j} \bar{q}_L^i H_1 Y_{Q1}^{ij} Q_R^j - \sum_{i,j} \bar{Q}_R^i M_Q^{ij} Q_L^j - \sum_{i,j} \bar{Q}_L^i \tilde{\phi}_2 Y_{u2}^{ij} u_R^j - \sum_{i,j} \bar{Q}_L^i \phi_2 Y_{d2}^{ij} d_R^j \\
& - \sum_{i,j} \bar{\ell}_L^i H_1 Y_{L1}^{ij} L_R^j - \sum_{i,j} \bar{L}_R^i M_L^{ij} L_L^j - \sum_{i,j} \bar{L}_L^i \phi_2 Y_{2e}^{ij} e_R^j \\
& + (h.c.).
\end{aligned} \tag{4.1}$$

We can take M_Q and M_L as diagonal matrices without lose of generality by the transformation of the vector-like fermions. For simplicity, we assume that all components of M_Q and M_L are larger than the other mass parameters in the Yukawa terms, and integrate out the vector-like fermions. Then we obtain the effective Yukawa couplings for the SM fermions,

$$\begin{aligned}
\mathcal{L}^{Yukawa} \simeq & - \sum_{i,j} \bar{q}_L^i \tilde{\phi}_3 y_u^{ij} u_R^j - \sum_{i,j} \bar{q}_L^i \phi_3 y_d^{ij} d_R^j - \sum_i \bar{\ell}_L^i \phi_3 y_e^i e_R^i \\
& - \sum_{i,j} \bar{q}_L^i H_1 \tilde{\phi}_2 \left(Y_{Q1} M_Q^{-1} Y_{2u} \right)^{ij} u_R^j - \sum_{i,j} \bar{q}_L^i H_1 \phi_2 \left(Y_{Q1} M_Q^{-1} Y_{2d} \right)^{ij} d_R^j \\
& - \sum_{i,j} \bar{\ell}_L^i H_1 \phi_2 \left(Y_{L1} M_L^{-1} Y_{2e} \right)^{ij} e_R^j \\
& + (h.c.).
\end{aligned} \tag{4.2}$$

Now we can clearly see that this type of Yukawa couplings is categorized as the one in the type-III 2HDM with extra singlet scalar at low energy, after integrating out W' and Z' as we have done in Sec. 3. In the limit that $v_1 \gg v_2$ and h_1 is extremely heavy, we lead the condition (iii): $\lambda_4 \sim \lambda_5 \sim \lambda_6 \sim \lambda_7 \sim 0$.

Table IV: Example of the Z_2 charge assignment for various two-Higgs doublet model.

Z_2	H_1	H_2	ϕ_3	$Q_{L,R}$	$L_{L,R}$	q_L	ℓ_L	u_R	d_R	e_R
type-II	\pm	\pm	$-$	\pm	\pm	$+$	$+$	$-$	$+$	$+$
type-X	\pm	\pm	$-$	\pm	\pm	$+$	$+$	$-$	$-$	$+$
type-Y	\pm	\pm	$-$	\pm	\pm	$+$	$+$	$-$	$+$	$-$

4.2 Emergence of the type-II, -X, and -Y two Higgs doublet models with a discrete symmetry

The type-II, -X, and -Y 2HDMs are also generated effectively from the setup in Eq. (4.1) by controlling the Yukawa couplings. For example, if $y_d = y_e = Y_{2u} = 0$ are realized, the model behaves as the type-II 2HDM at low energy. A popular way to forbid unwanted terms is to assign a discrete symmetry. An illustrative Z_2 charge assignments for the three types are shown in Table IV. The Z_2 symmetry plays a role in controlling the Yukawa couplings of ϕ_3 , so that it is the same as the one in the ordinary 2HDMs. Besides, these assignments forbid κ term which is corresponding to the soft mass term in the two-Higgs doublet model. However, κ term is required to avoid spontaneous Z_2 symmetry breaking, and thus this Z_2 symmetry must be broken softly.

In the type-II, -X, and -Y 2HDMs which are emerged from this model, the CP-odd and the charged Higgs are automatically degenerated and the Higgs potential respects CP symmetry. However, since we introduce Z_2 symmetry to forbid unwanted Yukawa interactions, we can not address the origin of Z_2 symmetry as we did in the type-I 2HDM shown in Sec. 3.

4.3 Emergence of the type-II, -X, and -Y two Higgs doublet models with a global symmetry

There is another way to forbid unwanted terms without Z_2 symmetry. Instead of imposing Z_2 symmetry, we impose a global U(1) symmetry. In order to forbid some Yukawa interactions, the right-handed fermions, H_2 , and ϕ_3 have to be charged under this global symmetry. However, this means that the global symmetry is spontaneously broken by the Higgs VEVs, and predicts a NG boson whose decay constant is around the electroweak scale. In that case, the model is similar to the QCD axion model and already excluded.

To avoid the constraint, we have to extend our model, and introduce new gauge singlet scalar S . The charge assignment is given in Table V. Here $x_u \neq x_d$ is required to forbid some Yukawa

Table V: Examples of U(1) charge assignments to forbid some Yukawa interaction terms. Here $x_u \neq x_d$. We take x_S as $-(x_u + x_d)$ or $-(x_u + x_d)/2$ to lead the soft Z_2 breaking term at low energy.

U(1)	q_L	Q_L	ℓ_L	L_L	Q_R	L_R	u_R	d_R	e_R	H_1	ϕ_2	ϕ_3	S
type-II	0	0	0	0	x_S	x_S	x_u	x_d	x_d	$-x_S$	$-x_d$	x_u	x_S
type-X	0	0	0	0	x_S	x_S	x_u	x_u	x_d	$-x_S$	$-x_d$	x_u	x_S
type-Y	0	0	0	0	x_S	x_S	x_u	x_d	x_u	$-x_S$	$-x_d$	x_u	x_S

interaction terms. There are two choices for x_S . When $x_S = -(x_u + x_d)$ is satisfied, we can write down $\phi_3^\dagger H_1 \phi_2$, while $x_S = -(x_u + x_d)/2$ leads $\phi_3^\dagger H_1 \phi_2 S^*$. If x_S does not satisfy both, the soft Z_2 breaking term is not emerged at low energy.

Since this model is similar to the DFSZ axion model [32], we expect that the VEV of S is $\mathcal{O}(10^{11})$ GeV. Then the VEV of H_1 should be also as large as the VEV of S to reproduce the fermion masses. This naturally leads the decouplings of ρ^μ and h_1 . In addition, we can solve the strong CP problem in the type-II and -Y cases if we choose $x_S = -(x_u + x_d)/2$.

5 Summary

The structure of the SM gives some hints to the new physics behind the SM. One important prediction of the SM is very small flavor and CP violations, and another is small deviation of the ρ parameter, which is realized by the custodial symmetry. These aspects strongly constrain the extensions of the SM.

The 2HDMs are widely discussed as candidates for BSMs. In the analysis of the 2HDMs, there are three features to realize the above conditions: (i) softly broken Z_2 symmetry, (ii) CP invariant Higgs potential, and (iii) degenerated mass spectra for the custodial symmetry. These three features play a crucial role in forbidding flavor violating Higgs interactions and avoiding large contribution to the ρ parameter. Besides, they usually simplify setups and analyses. However, their origins are unclear, and they look artificial from a viewpoint of bottom-up approach.

In this paper, we have proposed a model with extended electroweak gauge symmetry, $SU(2)_0 \times SU(2)_1 \times U(1)_2$, to explain the origins of the three conditions. We have shown that the low energy behavior of the model is well described by the type-I 2HDM with the three features. We also have discussed the extension of the models to derive other types of the 2HDM, and shown that the extended models can also solve the strong CP problem, imposing a global U(1) symmetry.

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